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HEARTWOOD FUNCTIONS AND COMPATIBLE TAPER AND MERCHANTABLE VOLUME SYSTEMS FOR *Acacia mangium* Willd., IN TABASCO, MEXICO

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ABSTRACT

In the last 20 years in Mexico, 30 % of the plantations supported by the Comisión Nacional Forestal (National Forestry Commission, CONAFOR) were made for timber purposes and established in the states of Tabasco, Veracruz, and Campeche, in Mexico. The economic valuation and management of plantations for these purposes are optimized by estimating the timber volume by type of product. The objectives of this study were to fit three compatible systems of segmented equations of taper and merchantable outside-bark volume; identify functions of taper and cumulative volume of heartwood; and analyze the dendrometric types based on the system developed by Fang *et al*. (2000) for plantations of clonal trees of *Acacia mangium* Willd., located in Huimanguillo, Tabasco, Mexico. Ninety-one (91) trees (2, 3, 7 and 8 years old) were cut down and two data groups were formed: 60 trees for fitting, and 31 for validation. The stem volume per section was calculated using the overlapping bolts method. The selection of the models was made according to the classification of four goodness-of-fit statistics. The system developed by Fang *et al*. (2000) showed reasonable estimates with both groups of measurements. The heartwood volume function overestimated the average percentage of heartwood relative to the inside bark volume in 2-year-old trees. However, it estimated them logically for the other ages: 34.7 % at 3 years, 61.8 % at 7 years, and 67 % at 8 years. The ratio heartwood taper to inside bark taper decreased after reaching 70 % of height.

Keywords: *Acacia mangium*, commercial forest plantation, minimum commercial diameter, product distribution, heartwood proportions, dendrometric types.

INTRODUCTION

The species *Acacia mangium* Willd. is a fast-growing multipurpose legume tree native to Indonesia, Papua New Guinea and north-eastern Australia (Hegde *et al*., 2013). It is one of the most widely used trees in commercial forest plantations in Asia and other Pacific regions (Krisnawati *et al*., 2011). Under favourable conditions, *Acacia mangium* reaches 30 m in height and 50 cm in diameter (Hegde *et al*., 2013).

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As a crop, the species has a high growth rate, depending on age, site quality and planting distances, with diameter increases of 1.8 to 5.8 cm yr^{-1} during the first 5 years (Reyes *et al*., 2018) and heights of 15 m after 4 years in the most fertile soils of the Philippines (Harrison and Herbohn, 2001). Also, after 3 years in average sites of the Caribbean region of Colombia (Torres-Vélez and Del Valle, 2007).

Acacia mangium wood is classified as hard, extremely heavy, strong, resistant, and suitable for producing furniture, door and window lintels, as well as household items (Sharma *et al*., 2012). In the world, the value of the wood of some species such as *Acacia melanoxylon* R. Br. is given mainly by its proportion of heartwood, which is characterized by a dark brown coloration with clear differentiation from sapwood in terms of colour and a high natural durability (Monteoliva *et al*., 2012).

Estimating the volumetric stocks of a commercial forest plantation for timber purposes allows the evaluation of its productive efficiency (Torres and Magaña, 2001). The functions of taper (d) and cumulative merchantable volume (Vc) are mathematical expressions that make it possible to accurately quantify the volumes and distribution of products derived from standing trees; if these models are defined by a common geometric structure and by the same parameters, there is a compatible volume-taper system (Cruz-Cobos *et al*., 2008).

The complexity of the mathematical formulations, as well as the number of variables and coefficients involved, require the classification of the taper models into simple, segmented, potential, exponential, variable exponent, trigonometric and logarithmic polynomials (Quiñonez-Barraza *et al*., 2014).

The heartwood is a portion of the wood that is between the pith and the sapwood; it is biologically inactive, it fulfils the function of support, and in general it has a more compact structure and a darker colour than the sapwood (Giménez *et. al*, 2000). The heartwood proportion is favoured by silvicultural practices that allow rapid individual growth (DeBell and Lachenbruch, 2009).

In Mexico, the forest plantation company Agropical S.A. de C.V. started a project for timber plantations of *Acacia mangium* of clonal origin 10 years ago. However, the work related to the analysis of the species growth and its proportion of heartwood is still scarce or private domain. The objective of the study was to fit three compatible systems of segmented equations of taper and merchantable volume, identify functions for estimating taper and accumulated volume of heartwood, and perform an analysis of the dendrometric types from the model of Fang *et al*. (2000) for *Acacia mangium* clone trees planted for commercial purposes in Huimanguillo, Tabasco. The hypothesis of this study is that segmented-type biometric models adequately describe the behaviour of taper and commercial volume of *Acacia mangium* trees of clonal origin, and the behaviour of taper and cumulative volume of heartwood is related to age and the height of the trees.

MATERIALS AND METHODS

Description of the study area, sample and dasometric variables

The study was developed in commercial forest plantations established with *Acacia mangium* cloned trees in Huimanguillo, Tabasco, Mexico. The soil is Feozem, the average annual temperature is 26 °C and the average annual rainfalls 2500 mm; typical characteristics of a warm humid climate with abundant rains in summer (Am) (INEGI, 2005).

The information used was obtained from 91 *Acacia mangium* trees of different ages (2, 3, 7 and 8 years old) from a destructive sampling carried out in 2017, where each tree was cut down and sectioned. Each one underwent the following measurements: normal diameter (D), diameter at different heights (d), bark thickness (gc), heartwood diameter at 1.3 m height (Dd), and heartwood diameter at different heights (dd), all these in centimetres. The measurement of the variables total height (H), height of the stump (hb), and height at different diameters (h) was done in meters.

The overlapping bolts method developed by Bailey (1995), based on the centroid method by Wiant *et al*. (1992), was used for the scale volume outside bark, inside bark and heartwood of each section; the cone volume formula was used for the tips.

The database was made up of 1372 pairs of height-diameter measurements. In order to detect atypical values and verify a logical behaviour of the variables, the variables of interest were audited graphically in their relative expression (Figure 1).

This database was divided randomly and without replacement into two groups with similar average characteristics. The data set designated for the fit of the models consisted of the measurements of 60 trees.

This amount was obtained with SAS® 9.0 (SAS Institute Inc., 2009) to have a sample with power of 95 %. The measurements of the remaining 31 trees were considered as an independent data set for the validation of the equations (Table 1).

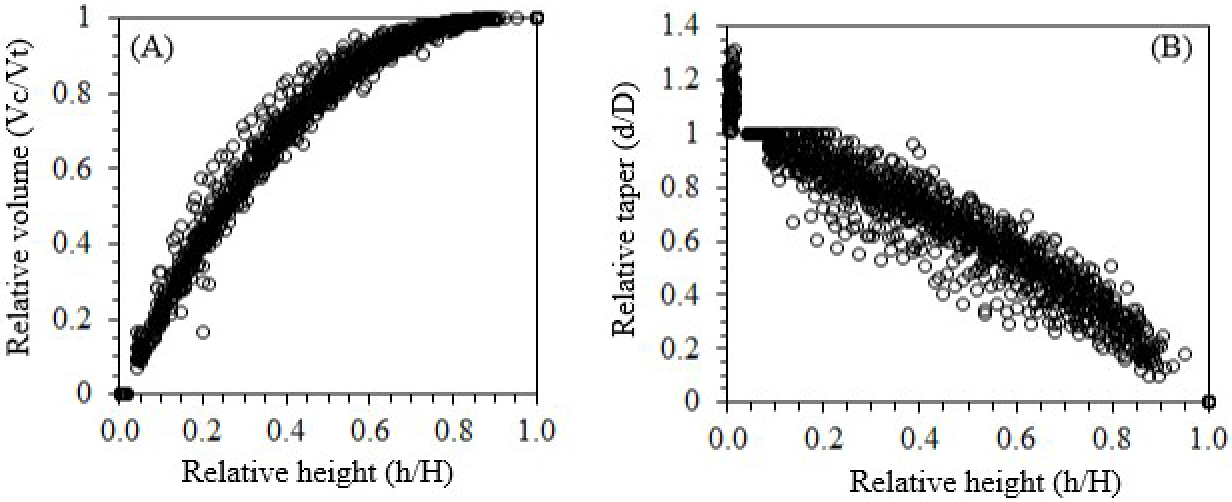

Figure 1. Logical behaviour of relative volume outside bark (A) and relative taper outside bark (B) with respect to relative height, where Vc: merchantable volume, Vt: total volume, d: taper, D: normal diameter, h: height at different diameters, H: total height.

Table 1. Descriptive statistics of the dasometric variables of the data groups used.

Variable	Fit data (n = 60)				Validation data (n = 31)			
	Min.	Average	Max.	DE	Min.	Average	Max.	DE
D (cm)	4.55	20.11	39.9	10.00	5.20	20.4	40.80	9.80
Dsc (cm)	4.15	18.42	37.7	9.22	5.00	18.75	38.00	8.97
Dd (cm)	0.00	14.08	32.35	9.81	0.00	14.71	31.7	9.38
H (m)	5.80	19.49	30.40	8.13	6.10	20.25	29.8	7.31
hb (m)	0.10	0.15	0.30	0.06	0.10	0.14	0.20	0.05
Vt (m^3)	0.01	0.39	1.05	0.35	0.01	0.41	0.97	0.36
Vtsc (m^3)	0.00	0.35	0.94	0.31	0.01	0.37	0.87	0.32
Vtd (m^3)	0.00	0.22	0.70	0.22	0.00	0.24	0.65	0.23
age (years)	2	6	8	3	2	6	8	2

n: number of observations, Min: minimum value, Max: maximum value, DE: standard deviation, D: normal diameter outside bark, Dsc: normal diameter inside bark, Dd: heartwood diameter at 1.3 m height, H: tree total height, hb: stump height, Vt: total volume outside bark, Vtsc: total volume inside bark, Vtd: total heartwood volume.

Compatible taper and merchantable volume systems

Three compatible taper and merchantable volume segmented systems, characterized by the estimation of two inflection points, were selected (Table 2).

The behaviour of the cumulative merchantable volume inside bark (Vsc) and of the taper inside bark (dsc) was expressed using direct proportion functions (a single parameter):

$$Vsc = \delta_0 \times Vc \tag{4}$$

$$dsc = \theta_0 \times d \tag{5}$$

where Vsc is the merchantable volume inside bark (m^3), Vc is the merchantable volume outside bark (m^3), dsc is the diameter inside bark (cm) at the height of h (m) from the height of the stump (hb), d is the diameter (cm) at the height of h (m) from the height of the stump (hb), and δ_0, θ_0 are the parameters to be estimated by the fit.

Heartwood functions

In order to adequately value the Acacia heartwood, the proportion and probable change of the heartwood in relation to taper (d) and merchantable volume outside bark (Vc) were modelled.

To explain the dynamics of the taper and the cumulative volume of the heartwood, functions involving the variables d, H and age were fitted because they are easy to obtain in the field (Table 3, Table 4).

Table 2. Compatible taper and merchantable volume systems.

Authors	Expression	System
Fang *et al*. (2000)	$d = C_1\left[H^{\frac{k-\beta_1}{\beta_1}}\left(1-q\right)^{\frac{k-\beta}{\beta}}\ alpha_1^{I_1+I_2}\ alpha_2^{I_2}\right]^{0.5}$ $\text{where}\begin{cases} I_1 = 1 \text{ if } p_1 \leq q \leq p_2; \text{ otherwise } 0 \\ I_2 = 1 \text{ if } p_2 \leq q \leq 1; \text{ otherwise } 0 \end{cases}$ $R_0 = \left(1-\frac{hb}{H}\right)^{\frac{K}{\beta_1}}\quad R_1 = \left(1-p_1\right)^{\frac{K}{\beta_1}}\quad R_2 = \left(1-p_2\right)^{\frac{K}{\beta_2}}$ $\beta = \beta_1^{1-(I_1+I_2)}\ \beta_2^{I_2}\ \beta_3^{I_2}$ $alpha_1 = \left(1-p_1\right)^{\frac{k(\beta_2-\beta_1)}{\beta_1\beta_2}}\quad alpha_2 = \left(1-p_2\right)^{\frac{k(\beta_3-\beta_2)}{\beta_1\beta_3}}$ $C_1 = \left[\frac{\alpha_0 D^{\alpha_1} H^{\alpha_2-\frac{k}{\beta_1}}}{\beta_1\left(R_0-R_1\right)+\beta_2\left(R_1-alpha_1\ R_2\right)+\beta_3\ alpha_1\ R_2}\right]$ $Vc = C_1^2\ H^{\frac{k}{\beta^1}}\{\beta_1 R_0 + \left(I_1+I_2\right)+\left(\beta_2-\beta_1\right)R_1 + I_2\left(\beta_3-\beta_2\right)$ $alpha_1\ R_2 - \beta(1\cdot q)^{\frac{k}{\beta}}\ alpha_1^{I_1+I_2}\ alpha_2^{I_2}]$	1
Max and Burkhart (1976)	$d = D\left[\beta_1\left(q-1\right)+\beta_2\left(q^2-1\right)+\beta_3\left(p_1-q\right)^2 I_1+\beta_4\left(p_2-q\right)^2 I_2\right]^{0.5}$ $\text{where}\begin{cases} I_1 = 1 \text{ if } q \leq p_1; \text{ otherwise } 0 \\ I_2 = 1 \text{ if } q \leq p_2; \text{ otherwise } 0 \end{cases}$ $Vc = kD^2H\Big\{\frac{\beta_2}{3}\left(q^3-z_1^3\right)+\frac{\beta_1}{2}\left(q^2-z_1^2\right)-\left(\beta_1-\beta_2\right)\left(q-z_1\right)$ $-\frac{\beta_3}{3}\left[\left(p_1-q\right)^3 J_1-\left(p_1-z_1\right)^3 K_1\right]$ $-\frac{\beta_4}{3}\left[\left(p_2-q\right)^3 J_2-\left(p_1-z_1\right)^3 K_2\right]\Big\}$ $\text{where}\begin{cases} J_1 = 1 \text{ if } q \leq p_1;\ J_2 = 0 \text{ if } q > p_2; \\ K_1 = 1 \text{ if } Z_1 \leq p_1;\ K_2 = 0 \text{ if } Z_2 > p_1; \end{cases}$	2
McClure and Czaplewski (1986)	$d = \left\{\left(\frac{\alpha_0 D^{\alpha 1} H^{\alpha 2}}{k/H}\right)\left[2\left(1-\beta_1\right)z+3\beta_1 z^2+I_1\beta_2\left(z-\gamma_1\right)^2+I_1\beta_3\left(z-\gamma_2\right)^2\right]\right\}^{0.5}$ $\text{where}\begin{cases} I_1 = 1 \text{ if } z \leq p_1; \text{ otherwise } 0 \\ I_2 = 1 \text{ if } z \leq p_2; \text{ otherwise } 0 \end{cases}$ $Vc = \left(\frac{\alpha_0 D^{\alpha_1} H^{\alpha_2}}{3}\right)\left[3+\left(3\beta_1-3\right)z^2-3\beta_1 z^3+\beta_2\left(1-\varphi_1\right)^3+\beta_3\left(1-\varphi_2\right)^3+I_2\beta_3\left(\varphi_2-z\right)^3\right]$	3

d: diameter outside bark (cm) at the height of h (m) from the height of the stump, D: normal diameter outside bark (cm), H: total height (m), hb height of the stump, Vc: merchantable volume outside bark (m^3), *k*: constant $\pi/40000$, α_i, β_i, φ_i, p_i are parameters to be estimated through the fit, I_i, J_1, K_1 are variables indicating dendrometric changes in the stem of the tree.

Table 3. Heartwood taper functions.

Expression	Model
$dd = \vartheta_0 \, d^{\vartheta_1} \, age^{\vartheta_2}$	6
$dd = \vartheta_0 \, d^{\vartheta_1} \, H^{\left(\frac{-\vartheta_2}{age}\right)}$	7

dd: heartwood diameter (cm) at height h (m) from stump height (hb), d: diameter outside bark (cm) at h height (m) from stump height (hb), H: total height (m), age (years), ϑ_i: parameters to be estimated through the fit.

Table 4. Functions of cumulative volume of heartwood (Vd).

Expression	Model
$Vd = \rho_0 \times Vc \times age^{\rho_1}$	8
$Vd = \rho_0 \times Vc \times H^{\frac{-\rho_1}{age}}$	9

Vc: merchantable volume outside bark (m^3), age (years), H: total height (m), ρ_i: parameters to be estimated through the fit.

Statistics and selection criteria

The evaluation of the models was based on the numerical analysis of common statistical criteria in the field of forest biometry: the root mean square error (RECM), the fitted coefficient of determination (R^2_{adj}), the mean absolute bias of the residuals ($|\bar{E}|$), and the coefficient of variation (CV) (Martínez-Ángel *et al.*, 2019):

$$RECM = \left[\frac{\sum_{i=1}^{n} \left(y_i - \hat{y}_i \right)^2}{n} \right]^{0.5} \tag{10}$$

$$R^2_{adj} = 1 - \left[\frac{(n-1) \sum_{i=1}^{n} \left(y_i - \hat{y}_i \right)^2}{(n-p) \sum_{i=1}^{n} \left(y_i - \bar{y} \right)} \right] \tag{11}$$

$$|\overline{E}| = \frac{\sum_{i=1}^{n} \left| y_i - \hat{y}_i \right|}{n} \tag{12}$$

$$CV = \left[\frac{\sum_{i=1}^{n} \left(y_i - \hat{y}_i \right)^2 / (n-1)}{\bar{y}} \right]^{0.5} \times 100 \tag{13}$$

where y_i, $\hat{y}_i$ and $\bar{y}_i$ are respectively the observed, estimated, and average values of the dependent variable (taper, merchantable volume), n is the total number of observations used in the fit, and p is the number of function parameters. In addition to the graphic analysis, to select the compatible taper and merchantable volume system, as well as the heartwood functions, which presented superior statistical criteria, the classification system proposed by Sakici *et al*. (2008) was used.

This system consists of classifying the fitted systems based on each statistical selection criterion, in such a way that the value of 1 corresponds to the best. Subsequently, the average per model is obtained, and the lower this value, the better its classification.

In order to assess the compatible systems as a whole, the statistics of both the taper model and the merchantable volume were classified and a single average per system was obtained.

Fitting strategy

Compatible systems and heartwood functions were fitted using the Full Information Maximum Likelihood (FIML) technique with the MODEL procedure of SAS® 9.0 (SAS Institute Inc., 2009). This technique allows the compatibility of the fit of the system and considers the simultaneous errors of the equation of taper and merchantable volume (Fang *et al*., 2000; Uranga-Valencia *et al*., 2015).

To ensure the evaluation of the partial derivatives that contain the logarithm of zero, a value of 0.001 was added when h = H and d = 0; thus, convergence problems and data loss were avoided (Fang *et al*., 2000).

The autocorrelation of the taper function of the compatible systems was corrected with the continuous autoregressive error structure of second order CAR (2) by means of the following structure (Zimmerman and Núñez, 2001):

$$e_{ij} = d_1\gamma_1^{h_{ij}-h_{ij-1}} e_{ij-1} + d_2\gamma_2^{h_{ij}-h_{ij-2}} e_{ij-2} + \varepsilon_{ij} \qquad (15)$$

where: $e_{ij} = j$ is the ordinary residual in tree i, $d_i = 1$ for $j > 1$ and $d_i = 0$ for $j = 1$, γ_i: are the autoregressive parameters to be estimated by the fit, $h_{ij} - h_{ij}$ is the separation distance from the *jth* to the *jth-1* observation within each tree, *hij > hij-i*, being ε_{ij} the error term under the independence condition.

The Durbin Watson (DW) statistic was used to corroborate the correction of the autocorrelation of the errors; the criterion indicates that there is no spatial or temporal dependence when the values are relatively close to 2 (Durbin and Watson, 1971; Barrios *et al*., 2014; Hernández-Ramos *et al*., 2017).

The heteroscedasticity problem associated with merchantable volume was corrected with the specification used by Quiñonez-Barraza *et al*. (2014): resid. Vc = resid. $Vc / [(D^2H)^w]^{0.5}$ for the SAS® 9.0 MODEL procedure (SAS Institute Inc., 2009), in which the power function of variance of the residual $\sigma_i^2 = (D^2H)^w$ (Crecente-Campo *et al*., 2009) is applied to correct, and the estimated unweighted model error $(\hat{e}_i)$ is used as the dependent variable in the model error variance $\left(\hat{e}_i^2 = w_0 (D^2 H)^{w1}\right)$.

Validation

The parameters obtained with the fit data of the selected compatible system were used to estimate the variables of interest with the validation data.

Since the heartwood is a part of the wood that is of commercial interest in other species such as *A. melanoxylon* and various *Eucalyptus* (Miranda *et al*., 2009; Igartúa *et al*., 2009), it was decided to estimate the taper and the cumulative volume of heartwood with all the functions proposed to analyze its validation.

The validation of the selected compatible system and the heartwood functions were compared by means of a *t* test for samples under the assumption of equal variances, with a confidence level of 95 %. Simultaneous fitting (with the fit sample data) of the compatible system of taper and merchantable volume outside bark was done, the respective functions inside bark, and the heartwood models selected as the most efficient.

The estimation of the variables of interest with the parameters of the models selected and validated in this study was compared graphically with the measurements of the validation data, and a random tree was selected to exemplify it.

Analysis of the dendrometric types from the model of Fang *et al*. (2000)

For the analysis of the dendrometric types, the starting point is a geometric body with a circular base defined by *d*, in cm, and *H*, in m, whose volume is described by the expression: $V = k \times \frac{\pi}{40\,000} \times d^2 \times h$, where *k* defines the type of solid described and indicates a transition towards some of the dendrometric types: *k* = 1/2 to a paraboloid, *k* = 1/3 corresponds to a cone, and *k* = 1/4 to a neiloid (Uranga-Valencia *et al*., 2015).

The scaling system of Fang *et al*. (2000) has the advantage of facilitating this analysis in each segment of the stem, since each section is represented as $\beta_i = k \times \frac{\pi}{40\,000}$ and, therefore $k = \beta_i = \times \frac{40\,000}{\pi}$.

RESULTS AND DISCUSSION

Compatible taper and merchantable volume systems

The values of the estimated parameters with their respective standard errors were significant ($p \leq 0.001$) in the compatible systems analyzed for clone trees of *Acacia mangium* (Table 5).

The goodness-of-fit statistics are included together with the rating given according to the classification system (Sakici *et al*., 2008), and the Durbin Watson (DW) statistic, which in all three systems gave values close to 2 (Table 6); therefore, the errors are independent (Durbin and Watson, 1971).

With the compatible systems developed by Fang *et al*. (2000) and Max and Burkhart (1976), the first inflection point was estimated at 10 % of the relative height, and the second at about 80 %.

This corresponds to a less cylindrical shape than the one considered with the inflection points obtained from the system developed by McClure and Czaplewski (1986), which occur at 15 and 91 % of the relative height.

Table 5. Parameters, estimators and standard errors of the parameters of the compatible systems analysed.

S		Estimators and standard errors										
		α_1	α_2	α_3	β_1	β_2	β_3	β_4	p_1	p_2	γ_1	γ_2
1	ψ	0.0001	1.6974	1.1047	0.00002	0.00003	0.00003		0.1054	0.8045	0.6766	0.6450
	ε	$1.1x10^{-6}$	0.0100	0.0127	$4.5x10^{-7}$	$2.9x10^{-7}$	$1.8x10^{-6}$		0.0048	0.0247	0.0211	0.0182
	∞	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001		<.0001	<.0001	<.0001	<.0001
2	ψ				-4.3981	2.1991	28.7961	-1.6441	0.0981	0.8550	0.7923	0.7139
	ε				0.8011	0.4215	5.0429	0.4013	0.0093	0.0337	0.0203	0.0145
	∞				<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001
3	ψ	0.0001	1.6272	1.2007	0.9752	-2.1227	46.0039		0.1486	0.9092	0.7391	0.6707
	ε	$2.6x10^{-5}$	0.0102	0.0126	0.0685	0.3966	14.0623		0.0464	0.0082	0.0221	0.0174
	∞	0.0001	<.0001	<.0001	<.0001	<.0001	0.0011		0.0014	<.0001	<.0001	<.0001

S: compatible system, ψ: estimator of the parameter, ε: standard error of the parameter, ∞: significance level of the parameter.

Table 6. Goodness-of-fit statistics of the compatible systems analysed and their rating (in parentheses).

System	Component	*RECM*	R^2 *adj*	$\lvert\bar{E}\rvert$	*CV*	Mean rating	*DW*
1	d	1.1488 (2)	0.9822 (2)	0.7799 (2)	7.7136 (2)	1.87	1.61
	Vc	0.0568 (2)	0.9648 (2)	0.0281 (1)	15.7841 (2)		
2	d	1.2059 (3)	0.9804 (2)	0.8492 (3)	8.0916 (3)	3	1.65
	Vc	0.0742 (3)	0.9399 (2)	0.0372 (3)	20.6387 (3)		
3	d	1.1258 (1)	0.9829 (1)	0.7715 (1)	7.5547 (1)	1.12	1.74
	Vc	0.0564 (1)	0.9652 (1)	0.0302 (2)	15.6895 (1)		

d: diameter outside bark (cm) at the height of h (m) from the height of the stump, Vc: merchantable volume outside bark (m^3).

The stem of *Acacia mangium* has the second inflection point defined at a relative height similar to other fast-growing species such as *Eucalyptus grandis* W. Hill ex Maiden and *E. dunni* Maiden; these plantations are located in Uruguay (Hirigoyen *et al*., 2021), where the first inflection point estimated with the system developed by Fang *et al*. (2000) occurs at 3 % of the relative height in both, and the second at 94 % and 85 % respectively; while, in the eastern Mediterranean region of Turkey, the *Eucalyptus camaldulensis* Dehn species recorded 6 and 71 % (Özçelik and Göçerí, 2015).

The compatible systems analyzed had efficient statistical criteria, with fitted determination coefficients of 98 % for the taper model and higher than 94 % for the merchantable volume model.

The system developed by McClure and Czaplewski (1986) had the best classification of statistical criteria followed by that of Fang *et al*. (2000), with almost imperceptible differences between both.

Heartwood functions

The estimated parameters of the functions of taper and cumulative volume of heartwood analyzed for clone trees of *Acacia mangium* (Table 7); the goodness-of-fit statistics (Table 8) are included with their classification (Sakici *et al*., 2008), as well as the DW statistic with which it was assumed that the errors were independent in both taper models because they presented values close to 2 (Durbin and Watson, 1971).

The estimated parameter values were significant ($p \leq 0.0001$) in all cases, which showed that the predicting variable relationships chosen for the taper and cumulative volume models were adequate to model the heartwood of *Acacia mangium*.

The classification of the statistical criteria of the taper functions indicated a tie, which is why function 7 was chosen for its logical qualities that allow its use with site indices when relating the variables of total height and age.

This poses a dynamic similar to that observed for *Eucalyptus globulus* Labill., in which the heartwood is related to the age and size of the tree. Consequently, trees with larger

Table 7. Parameters, estimators and standard errors of the parameters of the analysed heartwood taper and cumulative volume functions.

Function		Estimators and standard errors						
		ϑ_0	ϑ_1	ϑ_2	γ_1	γ_2	ρ_0	ρ_1
6	ψ	0.1893	1.2326	0.3160	0.6477	0.6900		
	ε	0.0067	0.0112	0.0133	0.0270	0.0246		
	∞	<.0001	<.0001	<.0001	<.0001	<.0001		
7	ψ	0.4651	1.2388	0.6412	0.6477	0.6888		
	ε	0.0210	0.0111	0.0287	0.0272	0.0248		
	∞	<.0001	<.0001	<.0001	<.0001	<.0001		
8	ψ						0.1717	0.6210
	ε						0.0356	0.0998
	∞						<.0001	<.0001
9	ψ						0.9284	0.9633
	ε						0.0683	0.1771
	∞						<.0001	<.0001

S: compatible system, ψ: estimator of the parameter, ε: standard error of the parameter, ∞: significance level of the parameter.

Table 8. Statistics of goodness of fit and qualification (in parentheses) of the functions of taper (dd) and cumulative volume (Vd) of heartwood analysed.

No.	Function	*RECM*	R^2 *adj*	$\lvert\bar{E}\rvert$	*CV*	Mean rating	DW
6	dd	1.0184 (1)	0.9835 (1)	0.7278 (2)	9.8718 (2)	1.5	1.81
7	dd	1.0209 (2)	0.9834 (2)	0.7137 (1)	9.5686 (1)	1.5	1.82
8	Vd	0.0229 (1)	0.9864 (1)	0.0159 (1)	10.5193 (1)	1	
9	Vd	0.0237 (2)	0.9854 (2)	0.0165 (2)	10.8873 (2)	2	

diameter and height will have a higher proportion of heartwood for the same age (Miranda *et al*., 2009).

The relationship of the variables Vc and age were able to logically explain the behaviour of the cumulative volume of heartwood, with statistical criteria slightly superior to function 9, therefore, the use of function (8) is considered. This is consistent with the relationship observed by Miranda *et al*. (2006) for *E. globulus*, which indicates that the portion of the trunk heartwood increases with the age of the tree; consequently, the volume of the heartwood is cumulative.

The average proportions of estimated heartwood volume (estimated Vd to observed Vc) with function 8 were 25.6 % at 2 years, 34.7 % at 3 years, 61.8 % at 7 years, and 67 % at 8 years.

When comparing them with the average proportions of the measurements of Vd observed in relation to Vsc observed, it was found that the function overestimates the proportion of Vd at 2 years since the real one was 5.6 %; however, the other ages had average proportions similar to those calculated: 32.2 %, 60.2 % and 64.5 % respectively. Function 9 also overestimated the proportion of Vd at the age of 2 years (32.2 %), but estimated proportions close to those observed at 3 years (41.2 %), at 7 years (64.5 %), and at 8 years (67.3 %).

The proportion of heartwood of *Acacia mangium* at 3 years is greater than that reported for *Eucalyptus grandis* at 4 years, according to the study by Winck *et al*., (2012), in which the proportion of heartwood volume in trees of *Eucalyptus grandis* was 31.2 % at the age mentioned, 52.3 % at 10 years and 76 % at 32 years.

Eucalyptus globulus at 9 y and *Acacia melanoxylon* at 19 to 32 y presented 37 and 43 % Vd with respect to Vt in plantations without silvicultural interventions after establishment in Buenos Aires, Argentina. However, these values only corresponded to the segment of the upper level of the stump to the base of the crown (Monteoliva *et al*., 2012). Whereas, in this study the heartwood from the base of the tree to the tip was measured.

The following proportions were calculated after excluding data from the 2-year-old group because estimates of cumulative volume and heartwood taper did not behave logically for this data set.

The average heartwood diameter in relation to the diameter inside bark observed was 88 % measured at 30 cm height and 79 % at 1.3 m. This ratio is 93 % with function 6, and 92 % with function 7 at 0.3 m height; while at a height of 1.3 m, both are 78 %.

The heartwood distribution in relation to relative height showed that the proportion of heartwood diameter decreases in the stem as relative height increases. At 50 % of H, the percentage of dd with respect to dsc is 68 % in agreement with the observed, and 70 % with the estimate with function 6, and 69 % with function 7. After 70 % of H, the proportions of heartwood taper with respect to dsc decrease; at this point the observed proportion was 62 %, and the estimated 63 %.

Validation

With the *t*-test for samples under the assumption of equal variances, the taper model of the system developed by McClure and Czaplewski (1986) obtained a two-tailed probability statistic of 0.004, and the volume statistic of 0.68, which indicated that the average of the volume estimates is not statistically different from zero, although the taper is. For this reason, the system developed by Fang *et al*. (2000) was selected for the estimation of Vc and d.

The averages of the estimates of the variables of interest with the system of Fang *et al*. (2000) and all four heartwood features had two-tailed probability statistics greater than 0.05.

The validation analysis of these models does not suggest statistical differences between the estimated and predicted values, therefore, we concluded that the models and estimated parameters are valid to apply them to independent data. Then, the simultaneous fit of these models was carried out and, although the statistical criteria did not abruptly improve, it had R^2 *adj* higher than 98 % for the d, Vc, dsc, dd and Vd models (Table 9).

The residuals of the selected models with respect to the predicted values of the fit data set are distributed logically according to the corrections made (Figure 2). The differences in the cumulative volume of heartwood are in amplitude ± 0.06 m^3 and those of its taper in ± 6 cm.

The residuals of dsc were distributed in amplitude ± 3 cm, and ± 4 cm^3 for Vsc; smaller intervals compared to the other taper models.

For the chosen tree, the taper showed a logical and expected behaviour both in the stem and in the heartwood; the estimated merchantable volume outside bark is greater than

Table 9. Statistics of goodness of fit, values and significance of the parameters of the selected models and fitted simultaneously.

Component	*RECM*	R^2 *adj*	DW	Parameter	Estimation	Standard error	Pr > |t|
d	1.1547	0.9820	1.54	α_0	0.0001	0.0000012	<.0001
Vc	0.0579	0.9634		α_1	1.7187	0.0106	<.0001
dsc	0.4473	0.9970		α_2	1.0849	0.014	<.0001
Vsc	0.0088	0.9990		β_1	1.7×10^{-5}	0.0000005	<.0001
dd	1.0233	0.9834	1.70	β_2	3.2×10^{-5}	0.0000003	<.0001
Vd	0.0229	0.9864		β_3	2.5×10^{-5}	0.0000014	<.0001
				p_1	0.1057	0.00496	<.0001
				p_2	0.7807	0.0227	<.0001
				γ_1	0.6534	0.0219	<.0001
				γ_2	0.6327	0.0193	<.0001
				γ_3	0.6053	0.0276	<.0001
				γ_4	0.6608	0.0283	<.0001
				ϑ_0	0.4735	0.0212	<.0001
				ϑ_1	1.2353	0.0115	<.0001
				ϑ_2	0.6573	0.0274	<.0001
				ρ_0	0.2073	0.0354	<.0001
				ρ_1	0.5298	0.0823	<.0001
				δ_0	0.9466	0.0006	<.0001
				θ_0	0.8942	0.0004	<.0001

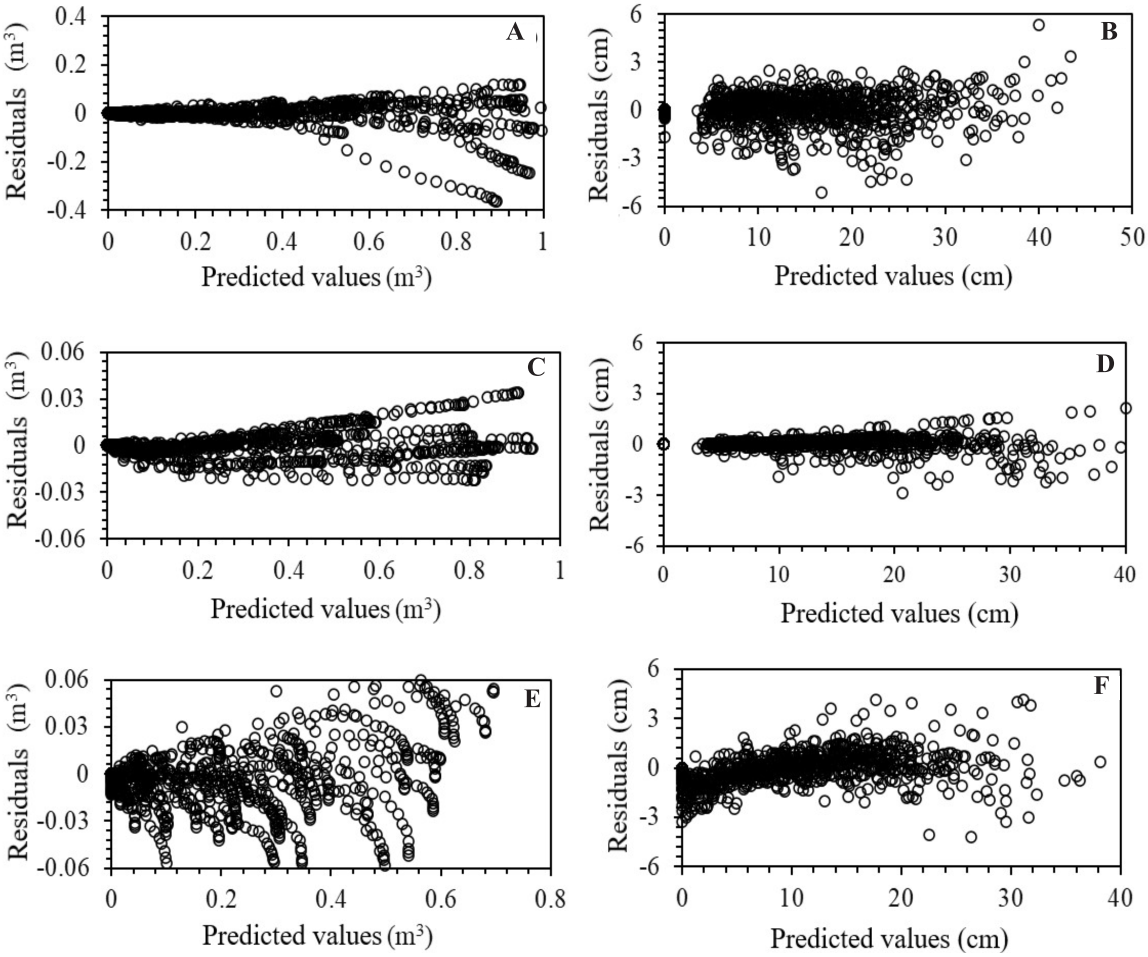

Figure 2. Distribution of the residuals of the merchantable volume outside bark (A), taper outside bark (B), merchantable volume inside bark (C), taper inside bark (D), cumulative volume of heartwood (E) and taper of heartwood (F) with respect to their predicted values.

the measured one. However, the estimated merchantable volume outside bark and taper inside bark are practically the same as those measured (Figure 3).

The analysis of the validation of the models with a set of independent measurements to those used for the estimation of the parameters is recommended to observe discrepancies between the expected logical behaviours, especially when fitting models that have not been widely used or studied, as was the case of heartwood functions.

Analysis of the dendrometric types from the system of Fang *et al.* (2000)

The system of Fang *et al.* (2000) described the stem of *Acacia mangium* as one composed of three segments. The predominant shape is that of a cone in transition to a paraboloid (k=0.41) and corresponds to the middle part. The first segment, from 0 to 11 % relative

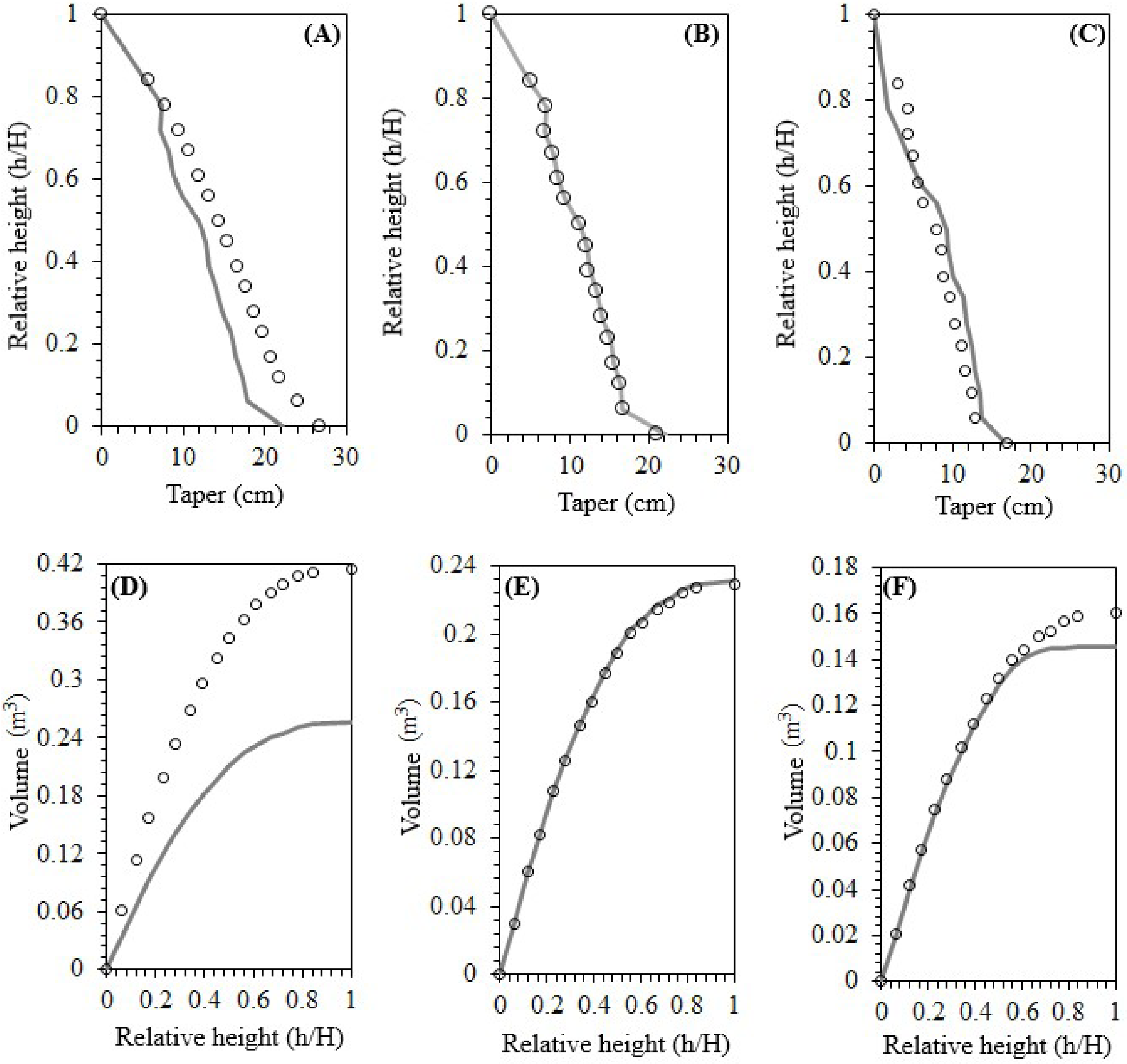

Figure 3. Estimates obtained (◦) with the parameters of the selected models of taper outside bark (A), inside bark (B), heartwood (C), merchantable volume outside bark (D), inside bark (E) and cumulative volume of heartwood (F) in regard to the data observed from the validation sample (-).

height, is described as a neiloid transitioning to a cone; while the third, from 80 to 100 %, is a conical formation.

Acacia mangium dendro-forms are similar to those described for species of the *Eucalyptus* genus, such as *E. grandis* and *E. dunni* (Hirigoyen *et al.*, 2021), set in plantations in Uruguay; while *E. camaldulensis* and *E. grandis* are in plantations in the eastern Mediterranean region of Turkey (Özçelik and Göçerí, 2015); in all of them, the middle segment of the stem is cone-shaped in transition to a paraboloid. This shows the effect of the management regimen and the genetic improvement that, by improving growth speed, sacrifices the shape of the stem; consequently, it takes a shape in transition towards more regular solids such as the paraboloid or the cylinder (Hernández-Ramos *et al*., 2017).

CONCLUSIONS

These plantations of *Acacia mangium* are of clonal origin and are under intensive management; therefore, the percentage of cumulative volume and the taper of heartwood observed in the measured trees showed the effect of genetic improvement and forestry operations, such as pruning and spacing between trees; in addition to presenting a shaft shape in transition towards a more regular solid such as the paraboloid.

The response to a management regimen that favours individual tree growth is a decreased stem taper and, consequently, an increase in volume and the amount of heartwood.

The steps to follow in subsequent research are to add data from trees of consecutive ages to increase the coefficient of fitted determination of the equation of merchantable volume outside bark; enrich the database to compare heartwood growth by age; as well as study the variables of the selected heartwood functions regarding the intensity of management and the site index.

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